REVENUE OF THE FIRM

Chapter Outline

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Chapter Summary

Appendix 2: Economics of Consumer Behavior

- I. Cardinal Utility Approach
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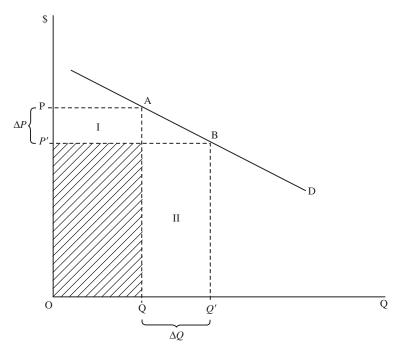
Questions

- 1. Total revenue (TR) is the total sales dollars that a firm takes in during some period of time, and it is found by multiplying the price of each item by the quantity sold. Average revenue is revenue per unit sold. It is found by dividing TR by quantity sold, and it will be equal to price as long as only one price is charged. Marginal revenue is the rate of change in TR with respect to a change in quantity sold. Point marginal revenue is given by $\frac{dTR}{dQ}$ and arc marginal revenue is given by $\frac{\Delta TR}{\Delta Q}$. Knowledge of the values of these variables is important to the firm when it is trying to determine the price which will maximize the profit obtained from the sale of a product. These matters will be discussed further in Chapter 7.
- 2. Roughly speaking, the price elasticity of demand measures the ratio of the percentage change in the quantity demanded of a product to the percentage change in its price. The point formula for E_p is $\frac{\partial Q}{\partial P} \times \frac{P}{Q}$. The arc formula for E_p is $\frac{Q_2 Q_1}{P_2 P_1} \cdot \frac{P_2 + P_1}{Q_2 + Q_1}$. When $|E_p| < 1$, a decrease in price will decrease total revenue, and vice versa.

When $|E_p| = 1$, a change in price will not change total revenue. When $|E_p| > 1$, a decrease in price will increase total revenue, and vice versa.

- 3. The determinants of demand are those variables other than a good's own price that affect the amount of the good buyers are willing and able to buy at some point in time. Some examples are income, prices of related goods, tastes, and advertising. Any change in one of the determinants of demand for a specific good or service will cause a shift in the demand curve for that good or service.
- **4.** One situation would be that in which a firm is trying to plan its output or inventory during the period preceding an expected recession or economic expansion. A second situation would be that in which a firm is trying to determine the long-run growth prospects for a particular product.
- 5. The factors would include the availability of related goods and their prices, consumer tastes and preferences, the extent to which the product is a necessity, level of income, and time span involved.
- **6.** The industry demand curve would generally be less elastic because of the availability of substitutes for the product of an individual firm.
- 7. Two goods are substitutes if one performs many of the same functions as the other, so that the consumer views one as a substitute for the other. Two goods are complements if having one good enhances the satisfaction which a person obtains from having the other. If the goods have a positive cross price elasticity of demand, they are substitutes. If the goods have a negative cross price elasticity of demand, they are complements. If the goods have a cross price elasticity of demand equal to zero, they are not related.
- **8.** Marginal revenue is zero when total revenue is at a maximum.

9.



In the figure, ΔTR is rectangle II *minus* rectangle I. Algebraically, for the change from point A to B on the demand curve, this is equal to $\Delta Q(P') + \Delta P(Q)$. Then $MR = \Delta TR/\Delta Q = P' + (\Delta P/\Delta Q)(Q) = P' - bQ$, where b is the absolute value of the slope of the demand curve. Let B approach A, so that P' approaches P. Of course, P = a - bQ is the equation for a straight-line demand curve (a being the price axis intercept and b the absolute value of the slope), and we can substitute this for P'. Thus, MR = (a - bQ) - bQ = a - 2bQ. Comparing the price equation with the MR equation, we have P = a - bQ and MR = a - 2bQ. The only difference between the two is that the slope of the demand curve is -b and that of MR is -2b.

Problems

1. a.

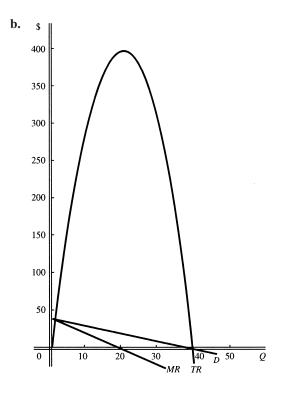
P	Q	TR	Arc MR
\$40	0	\$ 0	¢25
35	5	175	\$35
30	10	300	25
25	15	375	15
20	20	400	5
15	25	375	<u>-5</u>
10	30	300	-15
5	35	175	-25
0	40	0	-35

c. Between P = \$35 and P = \$30:

$$E_p = \frac{\Delta Q}{\Delta P} \cdot \frac{P_1 + P_2}{Q_1 + Q_2}$$
$$= \frac{5}{-5} \cdot \frac{.65}{15} = \frac{-4.33}{.15}$$

Between P = \$15 and P = \$10:

$$E_p = \frac{\Delta Q}{\Delta P} \cdot \frac{P_1 + P_2}{Q_1 + Q_2}$$
$$= \frac{5}{-5} \cdot \frac{25}{55} = \frac{-0.45}{25}$$



Demand is more elastic between P = \$35 and P = \$30 than between P = \$15 and P = \$10, since |-4.33| > |-0.45|.

- **2. a.** Barker's revenue will *increase* if the price of cement is lowered, because the demand for Barker's product is elastic with respect to price.
 - **b.** The new level of quantity demanded can be computed using the arc elasticity definition, which is as follows:

$$\begin{split} E_p &= \frac{\Delta Q}{\Delta P} \cdot \frac{P_1 + P_2}{Q_1 + Q_2} = -2 \\ &\frac{Q_2 - 10,000}{-1} \cdot \frac{5}{10,000 + Q_2} = -2 \\ &-5(Q_2 - 10,000) = -2(10,000 + Q_2) \\ &-5Q_2 + 50,000 = -20,000 - 2Q_2 \\ &-3Q_2 = -70,000 \\ &Q_2 = 23,333.33 \cong 23,333 \end{split}$$

The new level of total revenue will be $23,333 \times \$2 = \underline{\$46,666}$, compared to the original total revenue of \$30,000.

3. a.

P	Q	TR	Arc MR
\$120	0	\$ 0	\$105
105	10	1,050	
90	20	1,800	75
75	30	2,250	45
60	40	2,400	15
		- ´	-15
45	50	2,250	

b. Marginal revenue would equal price.Marginal revenue would be less than price.

4. a. Given
$$P_1 = \$1.40$$
, $Q_1 = 2,000$, $P_2 = \$1.20$, and an own price elasticity of -2.0 , the setup is

$$-2.0 = \frac{Q_2 - 2,000}{1.20 - 1.40} \cdot \frac{1.20 + 1.40}{Q_2 + 2,000}$$
$$= \frac{Q_2 - 2,000}{-.20} \cdot \frac{2.60}{Q_2 + 2,000}$$
$$-2.0 = \frac{Q_2 - 2,000}{-1} \cdot \frac{13}{Q_2 + 2,000}$$

Thus $13Q_2 - 26,000 = 2Q_2 + 4,000$; $11Q_2 = 30,000$, and $Q_2 = 2,727$.

b.
$$TR_1 = \$1.40(2,000) = \$2,800$$

$$TR_2 = \$1.20(2,727) = \$3,272.40.$$

TR increase is \$472.40.

5. a. Given
$$P_1 = \$90$$
, $Q_1 = 20,000$, $P_2 = \$80$, and an own price elasticity of -1.4 , the setup is

$$-1.4 = \frac{Q_2 - 20,000}{80 - 90} \cdot \frac{80 + 90}{Q_2 + 20,000}$$
$$= \frac{Q_2 - 20,000}{-10} \cdot \frac{170}{Q_2 + 20,000}$$
$$-1.4 = \frac{Q_2 - 20,000}{-1} \cdot \frac{17}{Q_2 + 20,000}$$

Thus $17Q_2 - 340,000 = 1.4Q_2 + 28,000$; $15.6Q_2 = 368,000$, and $Q_2 = 23,590$.

b.
$$TR_1 = \$90(20,000) = \$1,800,000$$

$$TR_2 = \$80(23,590) = \$1,887,200.$$

TR increase is \$87,200.

6. a.
$$\frac{Q_2 - 1,000}{-200} \cdot \frac{1,400}{Q_2 + 1,000} = -2$$

$$-7Q_2 + 7,000 = -2Q_2 - 2,000$$
$$-5Q_2 = -9,000$$

$$Q_2 = 1,800$$

b.
$$TR_2 = 1.800 \times \$600 = \$1,080,000$$

c. The corresponding increase in cost.

d.
$$\frac{Q_2 - 1,800}{800 - 900} \cdot \frac{1,700}{Q_2 + 1,800} = 0.5$$
$$-17Q_2 + 30,600 = 0.5Q_2 + 900$$
$$-17.5Q_2 = -29,700$$
$$Q_2 = 1,697$$

7. **a.**
$$\frac{Q_2 - 40,000}{-2} \cdot \frac{18}{Q_2 + 40,000} = -1.5$$
$$-9Q_2 + 360,000 = -1.5Q_2 - 60,000$$
$$7.5Q_2 = 420,000$$
$$Q_2 = 56,000$$

b.
$$TR_1 = \$10 \times 40,000 = \$400,000$$

 $TR_2 = \$8 \times 56,000 = \$448,000$
Increase in TR = \$48,000.

8. a. Total revenue will increase because demand is elastic; $|E_p| > 1$.

b.
$$E_p = \frac{\Delta Q}{\Delta P} \cdot \frac{P_1 + P_2}{Q_1 + Q_2} = -4$$

$$\frac{Q_2 - 1,000}{-1} \cdot \frac{9}{Q_2 + 1,000} = -4$$

$$-9(Q_2 - 1,000) = -4(Q_2 + 1,000)$$

$$-9Q_2 + 9,000 = -4Q_2 - 4,000$$

$$-5Q_2 = -13,000$$

$$Q_2 = \underline{2,600} \text{ units.}$$

9.
$$E_{yx} = \frac{\Delta Q_y}{\Delta P_x} \cdot \frac{P_{x1} + P_{x2}}{Q_{y1} + Q_{y2}} = 1.5$$

$$\frac{Q_{y2} - 10,000}{-2,000} \cdot \frac{22,000}{Q_{y2} + 10,000} = 1.5$$

$$-11(Q_{y2} - 10,000) = 1.5(Q_{y2} + 10,000)$$

$$-11Q_{y2} + 110,000 = 1.5Q_{y2} + 15,000$$

$$-12.5Q_{y2} = -95,000$$

$$Q_{y2} = \underline{7,600}.$$

10. a. Given $P_1 = \$1.89$, $Q_1 = 222,000$, $P_2 = \$1.65$, and an own price elasticity of -1.80 the setup is $-1.8 = \frac{Q_2 - 220,000}{1.65 - 1.89} \cdot \frac{1.65 + 1.89}{Q_2 + 220,000}$

$$= \frac{Q_2 - 220,000}{-0.24} \cdot \frac{3.54}{Q_2 + 220,000}$$
$$-1.8 = \frac{Q_2 - 220,000}{-1} \cdot \frac{14.75}{Q_2 + 220,000}$$

Thus $14.75Q_2 - 3,245,000 = 1.8Q_2 + 396,000$; $12.95Q_2 = 3,641,000$, and $Q_2 = 281,158$.

- **b.** New TR = \$1.65(281,158) = \$463,911. This compares with a previous TR of \$1.89(220,000) = \$415,800.
- For Mindy's, given a cross elasticity coefficient of +2.20 and an initial quantity of $Q_{M1} = 160,000$, and assuming they do not change their price when Charles's is cut from $P_{c1} = \$1.89$ to $P_{c2} = \$1.65$, the setup is

$$+2.2 = \frac{Q_{M2} - 160,000}{1.65 - 1.89} \cdot \frac{1.65 + 1.89}{Q_{M2} + 160,000}$$
$$= \frac{Q_{M2} - 160,000}{-.024} \cdot \frac{3.54}{Q_{M2} + 160,000}$$
$$2.2 = \frac{Q_{M2} - 160,000}{-1} \cdot \frac{14.75}{Q_{M2} + 160,000}$$

Thus, $14.75Q_{M2} - 2,360,000 = -2.2Q_{M2} - 352,000$; $16.95Q_{M2} = 2,008,000$, and $Q_{M2} = \underline{118,466}$.

d. For Mindy's, the change in TR is (118,466 - 160,000)(1.79) = -\$74,346.

11. a.
$$\frac{Q_2 - 25}{-20} \times \frac{180}{Q_2 + 25} = -1.5$$
$$-9Q_2 + 225 = -1.5Q_2 - 37.5$$
$$-7.5Q_2 = -262.5$$
$$Q_2 = 35$$

b.
$$\frac{Q_2 - 35}{-5} \times \frac{65}{Q_2 + 35} = 1.0$$
$$-13Q_2 + 455 = Q_2 + 35$$
$$-14Q_2 = -420$$
$$Q_2 = \underline{30}$$

c.
$$TR_2 = \$80 \times 30 = \$2,400$$

 $TR_1 = \$100 \times 25 = \$2,500$
 $\Delta TR = TR_2 - TR_1 = \$ -100$

12. a. For the given demand function and values of P_{ν} and I we obtain

$$Q = 2,000 - 50P + 40(30) + .01(40,000)$$
, so $Q = 3,600 - 50P$

- **b.** Transposing the result above, P = 72 .02Q. The demand curve is a straight line of the P = a bQ form, where a is the vertical axis intercept and b is the absolute value of the slope.
- **c.** $TR = P(O) = (72 .02O)(O) = 72O .02O^2$

d. As explained in the text without resort to the calculus, every straight line, downward-sloping demand curve will have a marginal revenue curve that is a straight line with *twice* the negative slope of the price or AR function. Therefore, from the answer to part b above, MR = 72 - .04Q.

- e. MR = 72 .04Q = 0 at Q = 72/.04 = 1,800. Or, more simply, since the MR curve is twice as steep as the AR curve and falls from the same vertical axis intercept, it intercepts the quantity axis at half the value that occurs where the AR intercepts that axis. Thus 3,600/2 = 1,800 is the value where MR = 0.
- **f.** Clearly, this will occur where MR = 0 at Q = 1,800. From the price equation, P = 72 .02(1,800) = 36, or one-half the value where both P and MR intercept the vertical axis. Thus,

$$TR = P(Q) = 36(1,800) = $64,800.$$

13. a. For the equation Q = 2,400 - 10P, AR = P = 240 - .1Q, and

$$TR = P(Q) = (240 - .1Q)(Q) = 240Q - .1Q^{2}$$

MR = 240 - .2Q (same vertical intercept and double the negative slope of AR.)

b. For the equation Q = 1,800 - 5P, AR = P = 360 - .2Q, and

$$TR = P(Q) = (360 - .2Q)(Q) = 360Q - .2Q^{2}$$

MR = 360 - .4Q (same vertical intercept and double the negative slope of AR.)

C1.
$$P = 120 - 1.5Q$$

$$TR = P \cdot Q = (120 - 1.5Q)Q = 120Q - 1.5Q^2$$

$$MR = \frac{dTR}{dO} = 120 - 3.0Q$$

$$AR = \frac{TR}{Q} = P = 120 - 1.5Q$$

C2. a.
$$Q_x = 197,000 - 100P_x + 50P_y + .025I + .02A + 10,000 P_L$$

$$Q_{\rm x} = 197,000 - 100(350) + 50(300) + .025(40,000) + .02(200,000) + 10,000(.30)$$

$$= 197,000 - 35,000 + 15,000 + 1,000 + 4,000 + 3,000$$

$$= 185,000$$

When $P_x = 400$, $Q_x = 180,000$, so we have:

$$E_p = \frac{\Delta Q_x}{\Delta P_x} \cdot \frac{P_{x1} + P_{x2}}{Q_{x1} + Q_{x2}}$$
$$= \frac{-5,000}{50} \cdot \frac{750}{365,000}$$
$$= -.21.$$

b. Inelastic; $|E_p| < 1$; decrease.

c.
$$E_I = \frac{\partial Q_x}{\partial I} \cdot \frac{I}{Q_x}$$
$$= .025 \left(\frac{40,000}{180,000} \right)$$
$$= \underline{.006}$$

The demand for Brand X washers is highly income inelastic, and the washers are barely normal goods. A 1 percent increase in income will result in only approximately a .006 percent increase in Q_x near this point of the demand function. This is an unusual result, since durable goods tend to be income elastic.

C3. a. Given the estimated demand curve, Q = 2840 - 20P, we have 20P = 2840 - Q and

(i)
$$AR = P = 142 - .05Q$$

(ii)
$$TR = Q(P) = 142Q - .05Q^2$$

(iii)
$$MR = dTR/dQ = 142 - 0.1Q$$

- **b.** Set MR = 0 = 142 0.1Q; Q = 1420. Here, price will be 142 71 = \$71. TR = P(Q) = \$71(1420) = \$100,820.
- c. Differentiating the Q equation, dQ/dP = -20. Thus point elasticity will be -20(P/Q). When Q = 1600, P = 142 .05(1600) = 142 80 = \$62. Therefore, the elasticity is -20(62/1600) = -0.78. Demand is inelastic since the absolute value of the elasticity coefficient is less than 1.0.
- **d.** Given $Q_1 = 1,000$ and $Q_2 = 1,100$, the AR equation can be employed to determine that $P_1 = 142 .05(1000) = 92$ and $P_2 = 142 .05(1100) = 87$. To find the elasticity, the setup is

$$E_p = \frac{1100 - 1000}{87 - 92} \cdot \frac{87 + 92}{1100 + 1000}$$

$$=\frac{100}{-5}\cdot\frac{179}{2100}=\frac{-1.70}{-1.70}$$

The result tells us that in the range from Q = 1,000 to Q = 1,100, demand is elastic, and a one-percent price cut will lead to a quantity increase of approximately 1.7 percent. If Alpha cuts price from \$92 to \$87, its TR will increase from \$92(1000) = \$92,000 to \$87(1100) = \$95,700.

$$\begin{aligned} \textbf{C4. a.} & \quad Q_T = 200 - .01P_T + .005\,P_M - 10P_g + .01\,l + .003A \\ & = 200 - .01(25,000) + .005(20,000) - 10(1.00) + .01(15,000) + .003(10,000) \\ & = 200 - 250 + 100 - 10 + 150 + 30 = 220 \\ & \quad E_p = \frac{\partial Q_T}{\partial P_T} \cdot \frac{P_T}{Q_T} = -.01\left(\frac{25,000}{220}\right) = \underline{-1.13}. \end{aligned}$$

- **b.** Elastic; the $|E_n|$ is greater than 1.
- **c.** At $P_M = \$20,000$, $Q_T = 220$.

At
$$P_M = $22,000, Q_T = 230$$
.

$$E_{TM} = \frac{\Delta Q_T}{\Delta P_M} \cdot \frac{P_{M2} + P_{M1}}{Q_{T2} + Q_{T1}} = \frac{10}{2,000} \cdot \frac{42,000}{450} = \underbrace{.47}_{...}$$

- **d.** Substitutes; the cross price elasticity is greater than zero.
- C5. a. $Q_L = 3,536 .5(10,000) + .2(8,000) + .008(45,000) + .0001(40,000)$ = 3,536 - 5,000 + 1,600 + 360 + 4 = 500 $E_I = \frac{\partial Q_L}{\partial_I} \cdot \frac{I}{Q_L} = .008\left(\frac{45,000}{500}\right) = \underline{0.72}.$
 - **b.** Normal; $E_I > 0$.
 - **c.** A 1 percent increase (or a 1 percent decrease) in income will result in approximately a .72 percent increase (or a .72 percent decrease) in the quantity demanded of lots.

d.
$$E_p = \frac{\partial Q_L}{\partial P} \cdot \frac{P_L}{Q_L} = -.5 \left(\frac{10,000}{500} \right) = \underline{\underline{-10}}.$$

e. Elastic; $|E_p| > 1$.

C6. a. At this point,

$$Q_s = 89,830 - 40(9,000) + 20(9,500) + 15(10,000) + 2(15,000) + .001(170,000) + 10(160) = 101,600$$

$$E_p = \frac{\partial Q_s}{\partial P_s} \times \frac{P_s}{Q_s} = \frac{-40(9,000)}{101,600} = \underline{\frac{-3.54}{1000}}.$$

- **b.** Elastic, since the absolute value of E_n is > 1.
- c. Smooth Sailing can tell that if it lowers the price of its sailboat, total revenue will increase as a result, since the answer in part (a) means that a 1 percent decrease in price will result in an increase of approximately 3.54 percent in quantity demanded. However, the firm still needs to calculate the increase in cost that would occur at the higher level of sales to determine if the price cut would increase total profit.

d.
$$E_{sx} = \frac{\Delta Q_s}{\Delta P_x} \times \frac{P_{x2} + P_{x1}}{Q_{s2} + Q_{s1}} = \frac{10,000}{500} \times \frac{19,500}{213,200} = \underline{1.83}.$$

Substitutes, since the cross price elasticity of demand is greater than zero.

e.
$$E_I = \frac{\partial Q_s}{\partial I} \times \frac{I}{Q_s} = 2 \times \frac{15,000}{101,600} = \underline{0.295}.$$

The boats are a normal good since the income elasticity of demand is greater than zero.

C7. a.
$$Q_x = 1420 - 20P_x - 10(40) + .02(8,000) + .04(1,200)$$

= $1420 - 20P_x - 400 + 160 + 48$
 $Q_x = 1,228 - 20P_x$.

- **b.** Product Y is a complement of X since the coefficient of P_y is negative; therefore the cross price elasticity of demand for Product X with respect to the price of Product Y will be less than zero.
- **c.** When $P_x = 50$, $Q_x = 1,228 (20)(50) = 228$. $E_p = (dQ/dP)(P/Q) = (-20)(50/228) = \underline{-4.39}$.

d.
$$Q_x = 1,228 - 20P_x$$
, so
$$-20P_x = Q_x - 1,228.$$

$$P_x = -.05Q_x + 61.4.$$

$$TR = P(Q) = -.5Q_x^2 + 61.4Q_x.$$

TR will be at a maximum where (dTR/dQ) = 0:

$$(dTR/dQ) = -.1Q_x + 61.4 = 0$$
$$-.1Q_x = -61.4.$$
$$Q_x = 614$$

Price will be equal to -.05(614) + 61.4 = -30.7 + 61.4 = \$30.70.

Total revenue will be equal to P(Q) = \$30.70(614) = \$18,849.80.

C8. a.
$$Q_{ACE} = 270 - .8(600) - 3(40) + .4(500) + .006(50,000) + .03(1,000)$$

= $270 - 480 - 120 + 200 + 300 + 30$
= 200
 $E_p = -.8(600)/200 = \underline{-2.4}$

b. Elastic, since the absolute value of E_p is greater than one. With a value of $E_p = -2.4$, a one percent change in price will result in approximately a 2.4 percent change, in the opposite direction, of the quantity demanded of ACE digital cameras.

c.
$$E_M = -3(40)/200 = -0.6$$

ACE digital cameras and the memory cards are complements, since the cross elasticity of demand is less than zero. A one percent change in the price of memory cards will result in approximately an 0.6 percent change, in the opposite direction, in the quantity purchased of ACE cameras.

d.
$$E_C = .4(500)/200 = \underline{1.0}$$

The two goods are substitutes, since the cross elasticity is greater than zero.

e.
$$E_I = .006(50,000)/200 = 1.5$$

Since the income elasticity of demand is greater than one, the digital cameras are normal goods. They can also be called *cyclically normal goods*, since Q_{ACE} varies more than proportionally with income. The income elasticity of 1.5 indicates that a one percent change in income will result in a 1.5 percent change, in the same direction, in quantity purchased of the digital cameras.

f.
$$Q_{ACE} = 270 - 120 + 200 + 300 + 30 - .8P_{ACE}$$

 $Q_{ACE} = 680 - .8P_{ACE}$

g. We first write the demand curve with price as the dependent variable:

$$-.8P_{ACE} = Q_{ACE} - 680$$

 $P_{ACE} = 850 - 1.25Q_{ACE}$

Total revenue is then obtained by multiplying the price equation by quantity, or

$$TR = 850Q_{ACE} - 1.25Q_{ACE}^{2}$$

 $MR = dTR/dQ = 850 - 2.5Q_{ACE}$